**Introduction of ARCH**

**Auto-Regressive Conditionally Heteroskedasticity (ARCH)**

Suppose, yt = ϴ Xt‘ + εt (1)

Ε t = σt zt , where zt ~ NID (0,1)

zt is a standard normal shock. Every line the shocks (zt is scalled with σt which is a stochastic process)

σt2 = ω + α εt-1 2 **--------- ARCH (2)**

σt2 = conditional variance of εt

ω = constant and is +ve

α >= 0

the equation (2) is a ARCH (1) model and can be extended to ρth order. That is

σt2 = ω + α1 εt-1 2 + α2 εt-2 2 + ----------- + αρ εt-ρ 2 **---------3**

**Analysis of ARCH model**

**1** zt ~ NID (0,1) => standard normal shocks

Ε t = σt zt => scaling of the shocks by σt

E ( εt 2 / It-1 ) => variance of the shocks conditional on the information set It-1

Where It-1 = **{** y1 x1 , yt-1 xt-1 **} = {** ε1 , εt-1**}**

E ( εt 2 / It-1 ) = E (σt 2zt 2 / I t-1 ) = σt2 E ( zt 2/ It-1 )

Since E ( zt 2/ It-1 ) = E ( zt 2 )

That is the conditional expectation E ( zt 2/ It-1 ) is equal to the unconditional expectation E ( zt 2 ) because there is no more information left in the information set ( It-1 ) for zt ,

Hence E ( zt 2 ) = 1

* E ( εt 2 / It-1 ) = σt2

This is how the conditional variance σt2 is defined based on the conditional expectation of the information set ( It-1 ) , from the part.

* E ( εt  / It-1 ) ~ N ( 0 , σt2 )

Is the conditional distribution of εt given It-1

**2** Define**:** εt 2 = E ( εt 2 / It-1 ) + Vt

we know E ( εt 2 / It-1 ) = σt2

Vt is a surprise term, independent to the information set, hence E ( Vt / It-1 ) = 0

* εt 2 = σt2 + Vt **-------- 5**

**or**

σt2 = εt 2 - Vt **---------6**

Rewriting equation (2)

Now in operating equation (a) we can re-write equation (2) as

…….. (7)

Equation (7) is a AR (1) process for squared innovation This could have been rewritten for an AR(P) process and for in the above manner to get a Squared innovation for AR(P) process.

3) Since, we known that the AR (1) Process is stationary, it is expected the , but here, we have put the restriction that

If , there exist a stationary solution.

= E ( = that means, we have constant unconditional variance of if

Let's take an example of and it confidence intervals.

Note that the ARCH model is conditionally heteroskedastic but unconditionally Homoskedastic. If we have large innovation shocks to last period innovation (t-1), then the conditional variance of the innovation will be large and if the innovation shock to (t-1) is small, then the conditional variance will be small.